Neatness is an asset in Algebra

1. Operations and Expressions and Equations

The way to learn mathematics is by practice.
Clear writing will help you avoid mistakes.
Neatness is an asset in algebra.

Arithmetic operations	**(+) addition**	**(−) subtraction**	**(×) multiplication**	**(÷) division**	**Exponentia-tion** x^y
Operation signs	$(+)\cdot(+)=(+)$	$(+)\cdot(+)=(+)$	$(-)\cdot(-)=(+)$	$(+)\cdot(-)=(-)$	$(-)\cdot(+)=(-)$
Inclusion symbols	Parentheses =( )	Brackets=[ ]	$2\div[1+2]$	$\frac{1}{2}$ — vinculum	x -base,
Operation terms	$1+2$ sum	$1-2$ difference	2×3 product	$\frac{2}{3}$ quotient	y - power, exponent,degree

An expression is a collection of numbers (symbols), operation signs, and inclusion symbols.

Examples to remember:

$3-(-1)=3+1=4$, $x+(-y)=x-y$

$(3+2)\times2\neq3+2\times2$, $(x-y)\cdot3=3x-3y\neq x-3y$

$3\div2\cdot3=\frac{3}{2}\cdot3\neq3\div(2\cdot3)=\frac{3}{2\cdot3}$, $3\div(x\cdot y)=\frac{3}{x\cdot y}\neq3\div x\cdot y=\frac{3}{x}\cdot y$

- Like terms ($x+y+2x=3x+y$),
- common factor $2x+4yx=2x(1+2y)$
- $(x-3)(x+8)=0\Rightarrow x=2,-8\;\; x^2+8x-3x-24=0$
- Multiply $(x-3)(x+8)\Rightarrow$ factor $x^2+8x-3x-24$
- We can multiply two or more terms, but factoring is a little bit difficult to find.

REMEMBER:

$a^2-b^2=(a+b)(a-b)$ difference of two squares $\Rightarrow$

$(a+b)(a-b)=a^2-b^2$ product of two conjugate terms **(in backward)**

If $a=\sqrt{x}, b=\sqrt{y}\Rightarrow(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})=x-y$

$(x+a)^2=x^2+2xa+b^2$ squaring $\Rightarrow$

$x^2+2xa+b^2=(x+a)^2$ factoring **(in backward)**

2. Quadratic and Radical Equation and Absolute value

2.1 Absolute value

$$|a| = \begin{cases} a \text{ if } a \geq 0 \text{ positive number} \\ -a \text{ if } a < 0 \text{ negative number} \end{cases}$$

$|a| \geq 0$ always positive. $|-3| = |3| = 3$,

$a^2 = |a|^2 \Rightarrow \sqrt{a^2} = |a|$, $5^2 = (-5)^2 = 25 \Rightarrow \sqrt{(\pm 5)^2} = |\pm 5| = 5 \geq 0$,

2.2 Solving an equation

$|x| = 1 \Rightarrow x = \pm 1$, $|x| = -1 \Rightarrow x$ has no solution since $|x| \geq 0$ positive number

$\sqrt{(x+3)^2} = 25 \Rightarrow |x+3| = 5$ or $(x+3) = \pm 5$

REMEMBER:

1. $|x| \geq 0$ the absolute value (distance) is always positive number, $|-3| = |3| = 3 \geq 0$
2. $x^2 \geq 0$ the square (area of a quadrate) is always positive number
3. $\sqrt{x} \geq 0$ the square root (radical) is always positive number if $x \geq 0$
4. $x^2 = b \Rightarrow \sqrt{x^2} = |x| = \sqrt{b} \Rightarrow x = \pm\sqrt{b}$ here $\sqrt{b} \geq 0$, $x^2 = 1 \Rightarrow |x| = 1, x = \pm 1$
5. $\sqrt{25} = |\pm 5| = 5 \geq 0$
6. $x^2 = (-x)^2$ but $x \neq -x$
7. $\sqrt{4} = \sqrt{2^2} = \sqrt{(-2)^2} \neq -2$

3. Cartesian Coordinate system

- Ordered pair, x-axis, abscissa , (horizontal), x-intercept,
- coordinate, y-axis, ordinate, (vertical), y-intercepts, quadrants 1,2,3, 4,
- line, slope, slope-intercept form, horizontal, vertical lines,
- point slope form, parallel, perpendicular lines, intersection of lines,
- system of equations, solution, linear combination method

4. Functions

- Linear function, dependent, independent variables, domain, range, quadratic functions
- $f : A \to B$ $y = f(x)$, graph= $(x, f(x))$, $y = ax + b$, $y = ax^2 + bx + c$
- Value of the function at the point, one-to-one function, inverse of the function
- Trigonometric function, logarithm, exponential, and absolute functions

5. Prime numbers, factors, and exponentation

- Fundamental theorem of calculus (each integer has exactly one set of prime numbers)
- $144 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$ (in backward)

- Exponentation, base, power, exponent, degree, Linear function, dependent, independent variables, domain, range, quadratic functions

REMEMBER:

$a \cdot a \cdot \ldots \cdot a = a^x,\ a^0 = 1$

$x^a \cdot x^b = x^{a+b} = x^a \cdot x^b$ product of 2 powers with equal bases **(add)**

$(x \cdot y)^a = x^a \cdot y^a = (x \cdot y)^a$ power of product

$(x^a)^b = x^{a \cdot b} = (x^a)^b$ power of power **(multiply)**

$x^{-a} = \frac{1}{x^a} = x^{-a}$ negative power, fractional form

$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a} = \left(\frac{x}{y}\right)^a$ power of a quotient

$\frac{x^a}{x^b} = x^{a-b}$ quotient of two powers with equal bases **(subtract)**

- $(xy^2)^{-3} \cdot x^3 \cdot y^2 = x^{-3} \cdot y^{-6} \cdot x^3 \cdot y^2 = x^0 \cdot y^{-6+2} = y^{-4}$

6. *Greatest common divisor (GCD and Least common multiple (LCM)*

- **Greatest common divisor**, relatively prime, $10 = 2 \cdot 5, 21 = 3 \cdot 7 \Rightarrow$ GCF=1
- x^4y^6 and $x^6y^4 \Rightarrow$ GCF= x^4y^4
- Factoring by grouping, $x^2 + 2x + xy + 2y = x(x+2) + y(x+2) = (x+2)(x+y)$
- Polynomial, rational expression $\frac{x+1}{x+4}$
- **Least common multiple** $6 = 2 \cdot 3, 8 = 2 \cdot 2 \cdot 2 \Rightarrow$ LCM= $2 \cdot 2 \cdot 2 \cdot 3 = 24$ (not 48)
- **Least common denominator (LCD)**
- **Division of polynomials** $(x^2 - 1) \div (x+1) = (x-1)$
- **Domain of a fractional equation** $(x+1) \neq 0$**, extraneous solution**

7. *Inequalities*

- Open and closed interval, sketch the graph, solving equality
- Solution, union $A \cup B$, intersection $A \cap B$, and, or

REFERENCES

[1] Paul Foerster, Algebra I, Addison Wesley, 1999

[2] Larson Hostetler, Algebra and Trigonometry, *Houghton Mifflin*, 1997

[3] Stephen Wolfram, *http://mathworld.wolfram.com*