Operation, Errors and Good hints in Algebra

IF I REPEAT MY MISTAKES I WILL NOT BE IMPROVED

ORDER of NUMBER

1. ERRORS with Numbers and Letters:

Error and Unusual notation	Correct	Tips
$x3$	$3 \cdot x$	Numbers first, variable second
$-x \cdot 2$	$-2 \cdot x$	Order is important
$2 + x \neq 2x$	Leave as $2 + x$	

SIGN CHANGE

2. ERRORS with Signs: (plus sign, positive number +, minus sign, negative number -)

$(+) \cdot (+) = (+)$ $(+) \cdot (+) = (+)$ $(-) \cdot (-) = (+)$ $(+) \cdot (-) = (-)$

Error and Unusual notation	Correct	Tips
$a - (-b) \neq a - b$	$a - (-b) = a + b$	$(-) \cdot (-) = (+)$
$2 - (-2) \neq 2 - 2$	$2 - (-2) = 2 + 2 = 4$	minus and minus=plus
$2 + (-3) \neq 1$	$2 - 3 = -1$	Negative number
$2 \cdot (-3) \neq 6$	$2 \cdot (-3) = -6$	Negative number

PARETHESES

3. ERRORS without using Parentheses: (separate two operations, order in operations)

Error and Unusual notation	Correct	Tips
$a \cdot -b \neq a \cdot (-b)$	$a \cdot (-b)$	Always use Parentheses
$-2 - -2 \neq -2 - (-2)$	$-2 - (-2) = -2 + 2 = 0$	Always use Parentheses
$-2^2 \neq (-2)^2$	$-2^2 = -4, (-2)^2 = 4$	Always use Parentheses

$x \div y \cdot x \neq \frac{x}{y \cdot x}$	$x \div (y \cdot x) = \frac{x}{(y \cdot x)}$	Always use Parentheses
$-a/-b \neq (-a)/(-b)$	$(-a)/(-b)$	Always use Parentheses
$a^2 + ab + b^2 \neq (a+b)^2$	$a^2 + 2 \cdot ab + b^2 = (a+b)^2$	Completing square
$(2x+4)^2 \neq 2(x+2)^2$	$(2 \cdot (x+2))^2 = 4 \cdot (x+2)^2$	Always use Parentheses

ARITHMETIC OPERATIONS

4. ERRORS with 4 Arithmetic Operations: (addition, subtraction, multiplication and division)

$(+)$	$(-)$	$(\times)$	$(\div)$

Error and Unusual notation	Correct	Tips
$x - a \neq a - x$	$3-2=1, 2-3=-1$	Order is important
$2x = -x+1 \Rightarrow x \neq 1$	$2x = -x+1 \Rightarrow 3x = 1$	Chang signs
$\frac{2x+1}{2} \neq x+1$	$\frac{2x+1}{2} \neq x + \frac{1}{2}$	Divide all terms
$a \div \frac{c}{a} \neq c$	$a \div \frac{c}{a} = a \cdot \frac{a}{c} = a^2/c$	Change the positions

FRACTION

5. ERRORS with Fractions:

Error and Unusual notation	Correct	Tips
$\frac{a}{c} + \frac{b}{d} \neq \frac{a+b}{c+d}$	$\frac{a}{c} + \frac{b}{d} = \frac{ad+bc}{c \cdot d}$	Same denominator. Property for adding fractions
$\frac{5}{2} + \frac{2}{5} \neq \frac{5+2}{2+5}$	$\frac{5}{2} + \frac{2}{5} = \frac{25+4}{10}$	Least Common Denominator
$\frac{a+b}{a} \neq 1+b$	$\frac{a+b}{a} = 1 + \frac{b}{a}$	Same denominator.
$\frac{\frac{1}{2}}{3} \neq \frac{2}{3}$	$\frac{(\frac{1}{2})}{3} = \frac{1}{2} \div 3 = \frac{1}{2} \cdot \frac{3}{1} = \frac{3}{2}$	Use the property of dividing
$(1/2)x \neq \frac{1}{2x}$	$(1/2)x = \frac{1}{2} \cdot x = \frac{x}{2}$	Use the fraction form

EXPONENTS

6. ERRORS with Exponents:

Error and Unusual notation	Correct	Tips
$a^0 \neq 0$	$a^0 = 1$	It is exponent. Different operation
$x^2 \cdot x^4 \neq x^8$	$x^2 \cdot x^4 = x^{2+4} = x^6$	Add powers,
$(x^2)^4 \neq x^6$	$(x^2)^4 = x^{2\cdot 4} = x^8$	Power to power.
$xy^2 \neq (xy)^2$	$(xy)^2 = x^2 \cdot y^2$	
$(y-2x)^2 \neq y^2 - 2yx - 2x^2$	$(y-2x)^2 = y^2 - 4yx + 4x^2$	Quadrate
$\frac{x^6}{x^2} \neq x^{\frac{6}{2}} = x^3$	$\frac{x^6}{x^2} = x^6 \cdot x^{-2} = x^{6-2} = x^4$	
$x^6 - x^2 \neq x^4$	$x^6 - x^2 = x^2(x^4 - 1)$	Associative

SQUARE

7. ERRORS with Square, radical:

Error and Unusual notation	Correct	Tips
$x^2 < 0$	$x^2 \geq 0$	Quadrate is always positive
$\lvert x \rvert < 0$	$\lvert x \rvert \geq 0$	Absolute value is positive
$\sqrt{x} < 0$	$\sqrt{x} \geq 0$	Square root is always positive
$\sqrt{(-1)^2} = -1 < 0$	$\sqrt{x^2} = \lvert x \rvert \geq 0$	Absolute value is positive
$(-1)^2 = 1^2 \Rightarrow -1 = 1$	$\sqrt{(\pm 1)^2} = \lvert -1 \rvert = \lvert 1 \rvert = 1$	Absolute value is positive
$-1 \neq 1 \Rightarrow (-1)^2 \neq 1^2$	$(-1)^2 = (+1)^2$	
$\sqrt{2x^2} \neq 2x$	$\sqrt{2x^2} = \sqrt{2} \cdot \sqrt{x^2} = \sqrt{2} \cdot \lvert x \rvert$	
$(-x)^2 \neq -x^2$	$(-x)^2 = (-x) \cdot (-x) = x^2$	
$\sqrt{-2} \cdot \sqrt{-2} \neq \sqrt{(-2) \cdot (-2)} = 2$	$\sqrt{-2} \cdot \sqrt{-2} = i\sqrt{2} \cdot i\sqrt{2} = -2$	Complex numbers

CANCELLATION LAW

8. ERRORS with Cancellation:

Error and Unusual notation	Correct	Tips
$\frac{a+bx}{a} \neq 1 + bx$	$\frac{a+bx}{a} \neq \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$	Cancel common factors, not common terms
$\frac{1}{3} \cdot 2 \cdot 3 \neq 0.333 \cdot 2 \cdot 3$	$\frac{1}{3} \cdot 2 \cdot 3 = 2$	Cancel before multiplying

$\frac{50}{24}\cdot x\cdot\frac{4}{5}\cdot\frac{6}{10}=\frac{200}{120}x\cdot\frac{6}{10}$	$\frac{50}{24}\cdot x\cdot\frac{4}{5}\cdot\frac{6}{10}=x$	Factor and Cancel before multiplying

USEFUL NOTATIONS

9. USEFUL METHODS:

Error and Unusual notation	Correct	Tips
$x3$	$3\cdot x$	Numbers first, variable second
$\frac{x-2}{3}$	$\frac{1}{3}(x-2)$	Fractional coefficient before
ax^2+bx+c	$a(x^2+\frac{b}{a}x+\frac{c}{a})$	Factor out the leading coefficient
x^2+bx+c	$x^2+2\cdot\frac{bx}{2}+c$	Completing the square
$-\frac{1}{2}x^2+2x-3=0$	$x^2-4x+6=0$	Multiply. Fractional form is not good
$\frac{a}{a+2}$	$\frac{a}{a+2}=\frac{a+(2-2)}{a+2}=1-\frac{2}{a+2}$	Add and subtract same terms to simplify
$\frac{x^{-2}}{y^4}\cdot x^3=\frac{1}{x^2\cdot y^4}\cdot x^3$	$x^{-2}\cdot y^{-4}\cdot x^3$	Use negative power instead of fractional terms
$\frac{2}{\sqrt{(x-3)}}$	$2\cdot(x-3)^{-\frac{1}{2}}$	Use fractional power
$\frac{2}{(x^2-6x-7)}$	$\frac{2}{(x-7)(x+1)}$	Factoring
$\frac{x^2}{(x+1)^2}+\frac{3x}{x+1}=\frac{2}{3}$	$y=\frac{x}{x+1}, y^2+3y-\frac{2}{3}=0$	Substitute. Find the simple form

USEFUL FORMULAS

10. USEFUL FORMULAS:

Formulas	Example	Tips
$x^2-a^2=(x+a)(x-b)$	$x^2-144=(x+12)(x-12)$	Difference of 2 squares. Factor $3=2\cdot\frac{3}{2}$
$x^2\pm 2x\cdot a+a^2=(x\pm a)^2$	$x^2+2x\cdot 2+4=(x+2)^2$	Perfect square
x^2+xa+a^2 incomplete term	$x^2+xa+a^2\pm xa=(x+a)^2-xa$	Add and subtract
$x^3-a^3=(x-a)(x^2+xa+a^2)$	$x^3-64=(x-4)(x^2+4x+16)$	$64=4^3$