

Algebra and Trigonometry in General

1. *Real numbers and Letters* ($x, y, a, b, 1, 2$)

- Natural N , integers Z , rational Q , irrational I and real numbers
- $N \subset Z \subset Q \subset R$ and $R = Q \cup I$ (completion, extension, closure, subset and union)
- Real number line, origin, coordinate, open and closed interval, $(-\infty, 2] \cup [3, +\infty)$
- Distance and absolute value, $d(a, b) = |b - a| \geq 0$
- Sequence $\{a_n\}$, sums, series $\sum_{i=1}^n a_n$, arithmetic, geometric series

2. *Algebraic operations and expressions* ($(x + 2) \cdot (a - 2)$)

- variables (x, y, z -unknowns, letters), constants ($a, b, c, 2, 3$ -numbers)
- Algebraic expressions=Connection of numbers and letters using operations and symbols
- algebraic expressions=Numbers+ Letters +Operations with parentheses $2x \cdot (x + c)$
- algebraic operations (4 arithmetic operations: addition, subtraction, multiplication, division, and one exponentiation (repeated multiplication) operation)
- sum, difference, product, quotient, fraction, exponents ($a + b, a - b, a \cdot b, a / b, a^b$)
- closure, identity, inverse, commutative, associative, and distributive properties
- $a * 1 = a, a * a^{-1} = 1, a * b = b * a, a * b = b * a, (a + b) + c = a + (b + c), (a + b)c = ac + bc$
- fractions $\frac{a}{b}$, numerator a , denominator b
- cancellation and substitution law $a + b = c + b \Rightarrow a = c, a = c \Rightarrow a + b = c + b$

3. *Exponentiation operation and Polynomials* ($ax^2 + c$)

- Polynomials=Connection of algebraic expressions using operations (exponentiation)
- quadrate, square root (radical), and exponents (repeated multiplications)
- $a^2 \geq 0, \sqrt{a} = |a| \geq 0, a \cdot a \dots \cdot a = a^b$
- $a^0 = 1, \frac{1}{a^b} = a^{-b}, a^x \cdot a^y = a^{x+y}, (a^x)^y = a^{x \cdot y}, x^n = a \Rightarrow x = \sqrt[n]{a} = a^{\frac{1}{n}}$
- $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x^1 + a_0$, polynomial, $f(x) = ax^2 + bx + c$
- n -degree, $a_n \neq 0$ -leading coefficient, x -one variable
- roots, factoring, multiplying 2 terms, $P(x) = 0, (x - a)(x + b), x^2 + bx - ax + ab$
- $x^2 + 2xa + a^2 = (x + a)^2$ completing square
- $f(x) = ax^2 + bx + c = a(x - x_1)(x - x_2)$ factoring
- quotient, fractional expression $\frac{x^2+2}{x-1}$, domain $x - 1 \neq 0$

- **cancellation, substitution** $\frac{a \cdot b}{b \cdot c} = \frac{a}{c}$, $\frac{a}{c} = \frac{a \cdot b}{c \cdot b}$
- polynomial division, factors, remainder, rational zeros, solutions, fundamental theorem of Algebra, rational functions, operations, Least common denominator
- N-th root, radicals, radical function, exponential and logarithmic functions
- Binomial theorem, $(x + y)^n$, Pascal's triangle,

4. Equations and Functions ($2x + b = 3$, $y = f(x) = ax + b$)

- **Equation=Connections of two algebraic expressions**, $A = B$
- **Linear equation** $ax + b = 0$, solution, extraneous solution,
- Solving, moving to left, right sides, changing signs,
- systems of linear equations $Ax = B$, intersections, matrices, determinants, row, column, square, identity, inverse, triangle, augmented matrix, elementary row operations, interchanging rows, multiplying, adding to another row, Cramer's rule
- **Quadratic equation** $ax^2 + bx + c = 0$, leading coefficient, factoring, completing the square, perfect square, incomplete square, square root, quadratic formula, discriminant, 0,1,2 solutions, complex numbers, Vieta's formula
- **Nonlinear, radical, absolute value, trigonometric, exponential and logarithm equations**
 $\sqrt{x+3} = x + 2$, $|x - 1| = -2$, $\sin(x + 3) = 1$, $e^x = 2$, $\log_a x = 4$
- **Functions** ($y = f(x)$, $f : A \rightarrow B$), mapping, correspondence, relation, rule, input (domain), output (range), only one value, one-to-one correspondence
- Operations, compositions of functions ($f(g(x)) = (f \circ g)(x)$), inverse function ($f(f^{-1}(x)) = x$), symmetric axis, reflection of the graph, transformations, shifting,
- **Linear function** $y = f(x) = ax + b$, graph, line, intercept, solution
- **Quadratic function** $f(x) = ax^2 + bx + c$, roots, parabola, up and down graphs
- **Rational function** $f(x) = 2x/(x - 3)$, domain, zeros
- **Exponential function** (nonlinear, growth function) $y = f(x) = a^x$, $1 \neq a > 0$,
- **Logarithmic function** (inverse of exponential function) $y = \log_a x$, $a, x > 0$, $a \neq 1$
- $\log_a(x \cdot y) = \log_a x + \log_a y$, $\log_a(x/y) = \log_a x - \log_a y$, $\log_a x^n = n \log_a x$
- Base a , e - number (Euler), natural logarithm ($\ln x$), domain, range, composition, inverse

5. Coordinate systems and Graphs

- **Graphs=coordinate + function**
- Line coordinate, plane (Cartesian) coordinate, x - axis (horizontal, abscissa), y - axis (vertical, ordinate), coordinate x , ordered pair (x, y) , quadrants 1,2,3,4
- distance, Pythagoras theorem, graphs, intercepts, solutions
- line in a plane, slope ($m = (y_2 - y_1)/(x_2 - x_1)$), equation of a line (point-slope, slope-intercept, general equation) vertical, horizontal, perpendicular, parallel lines, circle equation,
- $ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ (second degree equation function in two variables), discriminant $D = b^2 - 4AC$, parabola ($D = 0$), ellipse ($D < 0$), hyperbola ($D > 0$), conic, conic section,

- graphs of a linear, quadratic functions (parabola), absolute value function, square root function
- symmetric axis, inverse graph, shifting, reflecting, stretching

6. *Polar (circle) Coordinate systems and Trigonometry*

- [*Trigonometry=Circle + triangle=polar coordinate+ plane coordinate system*](#)
- Circle, radius, origin, angle, coordinate, $x = r \cdot \cos \mathbf{a}$, $y = r \cdot \sin \mathbf{a}$, $\sin^2 x + \cos^2 x = 1$
- Pythagoras theorem, right triangle, hypotenuse, opposite, adjacent sides, quotient
- 6 trig functions: $\sin x$ (sine), $\cos x$ (cosine), $\tan x$ (tangent)
- CSC (cosecant), SEC (secant) and COT (cotangent)
- angle of rotation, measure of a angle (degree, radian)
- inverse functions (\arcsin , \arccos , \arctan ,)
- law of Sines, Cosines, graphs, solutions $a^2 + b^2 - 2ab \cos \mathbf{a} = c^2$
- trigonometric identities, solving equations,

[1] Zalman Usiskin and others, [*Geometry*](#), Addison Wesley, 1997

[2] Paul Foerster, *Algebra I*, Addison Wesley, 1999

[3] Larson Roland, *Algebra II*, *Houghton Mifflin*, 1998

[4] Larson Hostetler, *Algebra and Trigonometry*, *Houghton Mifflin*, 1997